Announcements

1) (Juiz next week on Thursday, Examweek following on Thursday

2) HW #3 due today

Continuity

 $If f: D \rightarrow R$ where D is a region in \mathbb{R}' or \mathbb{R}^3 (or \mathbb{R}^2), We say f is continuous at (a,b) or (a,b,c) in D if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b) (D in \mathbb{R}^2)$ $\lim_{x,y,z} f(x,y,z) = f(a,b,c) (Din IR)$

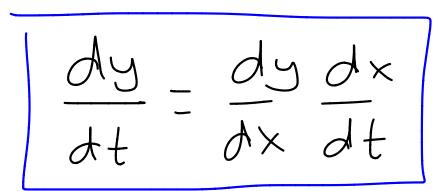
The Chain Rule (part 1)

Section 4.5

l'emember one-variable case :

If y=f(x) and x=x(t),

then

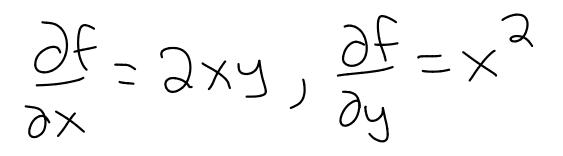


Provided all derivatives exist!

Example 1: Suppose f(x,y) = X'yand $X(t) = cos(t), y(t) = e^t$. Then we can compose to get a function g: IR > IR, $q(t) = f(cos(t), c^{t})$ = (os $(t)e^{t}$

lhen $g'(t) = -2\cos(t)\sin(t)e^t$ + $\cos^2(t)e^t$

Find in a different way.



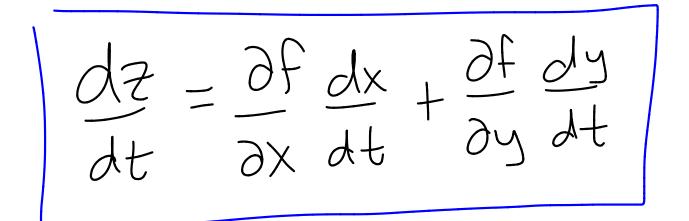
 $\chi'(t) = -\sin(t)$ $\chi'(t) = e^{t}$

Compute $\partial f(x(t), y(t)) \cdot dx$ λК + Of (x(t), y(t)) dy dt $= 2\cos(t)e^{t}(-\sin(t))$ + cosî(t)et = - 2 cos(t)sin(t)et + cosaltiet = g'(t)

Chain Rule (baby version)

If $7 = f(x_1y)$ is a function from Din R2 to R and

 $\chi = \chi(t), \quad \mathcal{Y} = \mathcal{Y}(t), \quad \text{then}$



If w = f(x, y, z) and X = X(t), Y = Y(t), Z = Z(t), $\frac{dw}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$

A similar formula holds for $g = f(x_1 x_2, ..., x_n)$ where $x_i = x_i (t)$ for all $1 \le i \le n$ $(f: D \rightarrow R, D in R^n)$

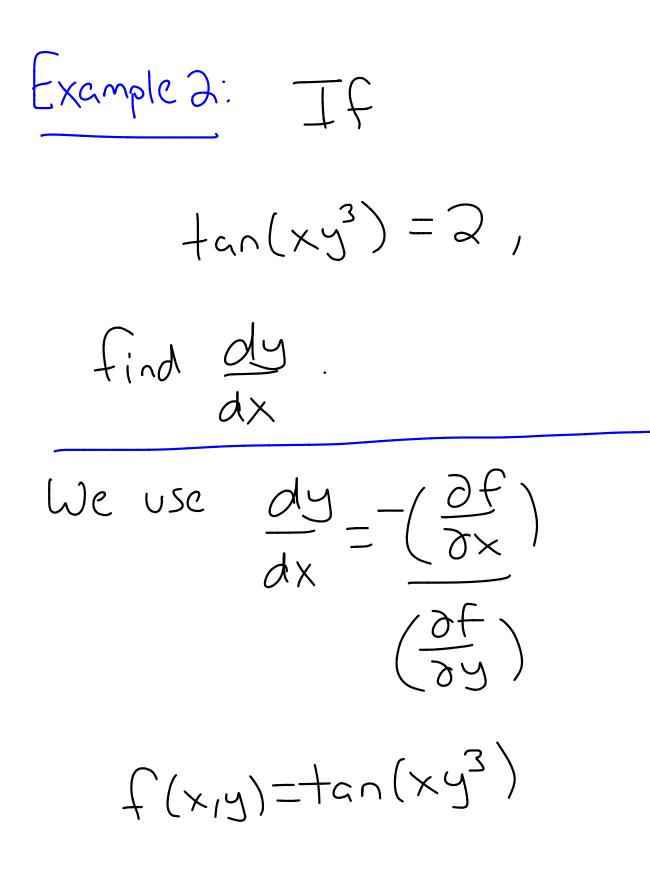
(onsequence: (implicit slopes)

Given an equation like $\chi^2 + \chi^2 = 1$, the graph is not always the graph of a function h: R > IR. However, the graph is a level curve of the function $f(x,y) = x^2 + y^2.$

Given an equation of the form f(x,y) = C, under certain conditions on f, the implicit function theorem Says we can solve for y implicitly as a function of X (on small enough open disks of the form $(X-G)^{2} + (Y-5)^{2} < \zeta)$

Regard y as a function of X, Consider q(x) = f(x, y(x)) = CTake the derivative of both Sides $O = \frac{d}{dx}(c) = \frac{d}{dx}(g(x))$ $= \frac{\partial f}{\partial x} \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \frac{\partial f}{\partial x} \frac{\partial y}{\partial y} \frac{\partial f}{\partial x}$ (by baby chain rule)

We get $O = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx}$ and so if $\frac{\partial F}{\partial y} \pm 0$, $= - \left(\frac{\partial f}{\partial x} \right)$ dy dx $\left(\begin{array}{c} \partial f \\ \partial y \end{array}\right)$



 $f(x,y) = \tan(xy^3)$

 $\frac{\partial f}{\partial x} = \operatorname{Sec}^{2}(xy^{3}) \cdot y^{3}$ $\partial f = \operatorname{Sec}^{2}(xy^{3})(3y^{2}x)$ $-\frac{Sec^{2}(xy^{3})y^{3}}{Sec^{2}(xy^{3})(3y^{3}x)}$ dy $-y^{3} = -\frac{y}{3x}$ $3y^{3}x$

Example 2: Suppose $f(x,y) = x^3 + \sqrt{y}$ and now instead of functions of one variable, $X = X(S,t) = \ln(S+t)$ $y = y(s,t) = s^{4}t^{8}$. We can make G(s,t) = f(x(s,t), y(s,t)),what are $\frac{\partial g}{\partial s}$, $\frac{\partial g}{\partial t}$?

 $g(s,t) = (ln(s+t)) + \sqrt{s^{4}t^{8}}$ $= \left(\left| n(s+t) \right|^3 + s^2 + s^4 \right)$

 $\frac{\partial g}{\partial s} = 3(\ln(s+t))^{2} + 2st^{4}$ Stt $\frac{3}{2} = \frac{3(\ln(stt))^2 + 4s^2t^3}{5tt}$